

## Statistics





## From the 2012 Administration

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Note: This publication shows the page numbers that appeared in the 2011–12 AP Exam Instructions book and in the actual exam. This publication was not repaginated to begin with page 1.

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## **Exam Instructions**

The following contains instructions taken from the *2011–12 AP Exam Instructions* book.

## **AP® Statistics Exam**

Regularly Scheduled Exam Date: Wednesday afternoon, May 16, 2012 Late-Testing Exam Date: Wednesday morning, May 23, 2012

## **Section I: At a Glance**

**Total Time:** 

1 hour, 30 minutes

**Number of Questions:** 

40

**Percent of Total Score:** 

50%

Writing Instrument:

Pencil required

**Electronic Device:** 

Graphing calculator expected

## **Section II: At a Glance**

**Total Time:** 

1 hour, 30 minutes

**Number of Questions:** 

6

**Percent of Total Score:** 

50%

**Writing Instrument:** 

Either pencil or pen with black or dark blue ink

**Electronic Device:** 

Graphing calculator expected

## Part A:

**Number of Questions:** 

5

**Suggested Time:** 

1 hour, 5 minutes

**Percent of Section II Score:** 

75%

Part B:

**Number of Questions:** 

1

**Suggested Time:** 

25 minutes

**Percent of Section II Score:** 

25%

## **Section I: Multiple Choice Booklet Instructions**

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

## **Section II: Free Response Booklet Instructions**

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

## What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- 2011-12 AP Coordinator's Manual
- This book *AP Exam Instructions*
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener

- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
  - "Exam in Progress"
  - "Cell phones are prohibited in the testing room"

Students are expected to bring graphing calculators to this exam. Before starting the exam, make sure no one has a laptop or other portable/handheld computer, pocket organizer, calculator with a QWERTY keyboard (e.g., TI-92 Plus, Voyage 200), electronic writing pad, or pen-input/stylus-driven device (e.g., Palm Pilot). Cell phone calculators are not allowed. If a student has a calculator that is not acceptable or has one that does not have statistical capabilities, you may provide one from your supply. See pages 40–42 of the *2011-12 AP Coordinator's Manual*. If the student does not want to use the calculator you provide, he or she must hand copy, date, and sign the release statement on page 41 of the *Manual*.

Students may have no more than two calculators on their desks, and calculators may not be shared. Students with Hewlett-Packard 48 Series graphing calculators may use cards designed for use with these calculators. Proctors should make sure Hewlett-Packard calculators' infrared ports are not facing each other. Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in cancellation of AP Exam scores.

## **SECTION I: Multiple Choice**

Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.

Make sure you begin the exam at the designated time.

If you are giving the regularly scheduled exam, say:

It is Wednesday afternoon, May 16, and you will be taking the AP Statistics Exam.

*If you are giving the alternate exam for late testing, say:* 

It is Wednesday morning, May 23, and you will be taking the AP Statistics Exam.

In a moment, you will open the packet that contains your exam materials. By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the 2011-12 Bulletin for AP Students and Parents. You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

Look at page 1 of your answer sheet and locate the dark blue box near the top right-hand corner that states, "Take the AP Exam label from your Section I booklet and place the label here.". . .

Now look at the front cover of your exam booklet and locate the AP Exam label near the top left of the cover. . . .

Carefully peel off the AP Exam label and place it on your answer sheet on the dark blue box that we just identified. . . .

Now read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name, and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

Section I is the multiple-choice portion of the exam. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled. Are there any questions? . . .

You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses on your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work. Calculators may be used for both sections of this exam. You may place your calculators on your desk. Are there any questions? . . .

You have 1 hour and 30 minutes for this section. Open your Section I booklet and begin.

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9,6,3	Note Start Time here	Note Stop Time here	
mark	ing their answers in pencil	on their answer sheets, and that	they are not looking at their
shrin	kwrapped Section II bookle	ets. Proctors should walk around	l and make sure Hewlett-Packard
calcu	lators' infrared ports are no	t facing each other. After 1 hour	and 30 minutes, say:

Stop working. Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. I will now collect your answer sheet.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. Then say:

Now you must seal your exam booklet. Remove the white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .

Check that each student has signed the front cover of the sealed Section I booklet. There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. Everything you placed under your chair at the beginning of the exam must stay there. Leave your shrinkwrapped Section II packet on your desk during the break. You are not allowed to consult teachers, other students, or textbooks about the exam during the break. You may not make phone calls, send text messages, check email, use a social networking site, or access any electronic or communication device. Remember, you are not allowed to discuss the multiple-choice section of this exam. Failure to adhere to any of these rules could result in cancellation of your score. Are there any questions? . . .



You may begin your break. Testing will resume at \_\_\_\_\_\_.

## **SECTION II: Free Response**

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished.  $\dots$ 

Now place an AP number label on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and read Item 1 under "Important Identification Information." Print the first two letters of your <u>last</u> name and the first letter of your <u>first</u> name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4....

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the information on the back cover of the exam booklet. Do not open the booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

Section II has two parts. You have 1 hour and 30 minutes to complete all of Section II. You are responsible for pacing yourself, and may proceed freely from one part to the next. You must write your answers in the exam booklet using a pen or a No. 2 pencil. If you use a pencil, be sure that your writing is dark enough to be easily read. If you need more paper during the exam, raise your hand. At the top of each extra piece of paper you use, be sure to write only your AP number and the number of the question you are working on. Do not write your name. Are there any questions? . . .

You may begin Part A.

Note Start Time here \_\_\_\_\_\_. Note Stop Time here \_\_\_\_\_. Check that students are writing their answers in their exam booklets. You should also make sure that calculators' infrared ports are not facing each other and that students are not sharing calculators. After 1 hour and 5 minutes, say:

There are 25 minutes remaining and you may want to move on to Part B, if you have not already started answering that question.

After 15 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, face up. . . .

If any students used extra paper for the free-response section, have those students staple the extra sheet/s to the first page corresponding to that question in their exam booklets. Then say:

## Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box, and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

*If you are giving the regularly scheduled exam, say:* 

You may not discuss these specific free-response questions with anyone unless they are released on the College Board website in about two days. You should receive your score report in the mail about the third week of July.

*If you are giving the alternate exam for late testing, say:* 

None of the questions in this exam may ever be discussed or shared in any way at any time. You should receive your score report in the mail about the third week of July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you.

Then say:

## You are now dismissed.

All exam materials should be put in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to view a separate AP Instructional Planning Report (for regularly scheduled exams only) or Subject Score Roster at the class section or teacher level. See "Post-Exam Activities" in the 2011-12 AP Coordinator's Manual.
- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

## Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

2012 Answer Sheet

your AP Student Pack and

NAME AND EXAM AREA — COMPLETE THIS AREA AT EVERY EXAM.

Take an AP Number label from place the label here.

Take the <u>AP Exam label</u> from your Section I booklet and place the label here.

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## **USE NO. 2 PENCIL ONLY**

ONLINE PROVIDER CODE

F. MULTIPLE-CHOICE BOOKLET  SERIAL NUMBER CODE										Print Form: Print Form Code:	COMPLETE THIS AREA ONLY ONCE.	M. EXPECTED DATE OF	COLLEGE ENTRANCE		Fall     2012	Winter/Spring 0 2013				N. STUDENT SEARCH SERVICE®	want the College Board to send	information about me to colleges, universities and government scholarship	programs interested in students like me.	G. PARENTAL EDUCATION LEVEL	Mother/ emale Guardian	Grade school	Some high school	High school diploma or equivalent	Business or trade school	Some college
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To maintain the security of the exam and the validity of my AP score, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after the completion of the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the 2011-12 Bulletin for AP Students and Parents.	including using testing accommodations (e.g., extended time, computer, etc.) <u>only</u> if I have been preapproved by Co for Students with Disabilities.	ı your college applications.		Legal First Name — F			B B B B B																			(S) (S) (S) (S) (S) (S) (S)				ETS USE ONLY
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Some graduate or professional school

Bachelor's or four-year degree

Associate or two-year degree

Other Hispanic, Latino or Latin American
White

Another language

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Option 2 (7)

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Other

Graduate or professional degree

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Section Number

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1) Option 1

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PAGE 3

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83		98 (A (B) (C) (D) (E)	113 (A) (B) (C) (D) (E)
84	ABCDE	99 A B © D E	114 (A (B) (C) (D) (E)
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By providing your email address, you are granting the College Board permission to use your email in accordance with the policies in the 2011-12 Bulletin for AP Students and Parents.

Y. EMAIL ADDRESS

# **Section I: Multiple-Choice Questions**

This is the multiple-choice section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

# PLACE SEAL HERE

## AP® Statistics Exam

## **SECTION I: Multiple Choice**

2012

## DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

## At a Glance

## **Total Time**

1 hour, 30 minutes

**Number of Questions** 

**Percent of Total Score** 

## Writing Instrument

Pencil required

## **Electronic Device**

Graphing calculator expected

## **Instructions**

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

## Sample Question

## Sample Answer

Chicago is a







- (A) state
- (B) city
- (C) country
- (D) continent
- (E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.



Form I Form Code 4IBP4-S Formulas begin on page 3. Questions begin on page 6. Tables begin on page 42.

## **Formulas**

(I) Descriptive Statistics

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \overline{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \overline{x}}{s_x} \right) \left( \frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \overline{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_X = \sum x_i p_i$$

$$Var(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu_{\chi} = np$$

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size n from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\overline{X}} = \mu$$

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

## (III) Inferential Statistics

 $Standardized \ test \ statistic: \frac{statistic - parameter}{standard \ deviation \ of \ statistic}$ 

Confidence interval: statistic ± (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} + \frac{p_2(1-p_2)}{n_2}$ Special case when $p_1 = p_2$
	$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Chi-square test statistic = 
$$\sum \frac{(observed - expected)^2}{expected}$$

## STATISTICS SECTION I

Time—1 hour and 30 minutes Number of questions—40

Percent of total score—50

**Directions:** Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. Of the following dotplots, which represents the set of data that has the greatest standard deviation?

(4)	*	*	*	*	*	*	*	*	*	*
(A)	0									10

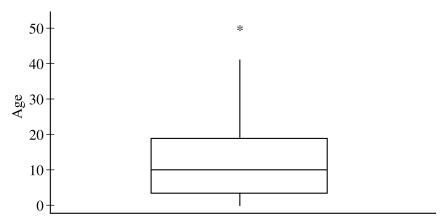
(B) 
$$\frac{*****}{0}$$
  $\frac{*****}{10}$ 

(C) 
$$\frac{***}{0}$$
 \*\*\*\* \*\*\*

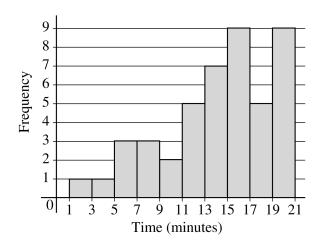
(D) 
$$\frac{****}{0}$$
 \*\* \*\*\*\*

(E) 
$$\frac{**}{0}$$
  $\frac{**}{0}$   $\frac{**}{10}$ 

2. A random sample of 374 United States pennies was collected, and the age of each penny was determined. According to the boxplot below, what is the approximate interquartile range (IQR) of the ages?



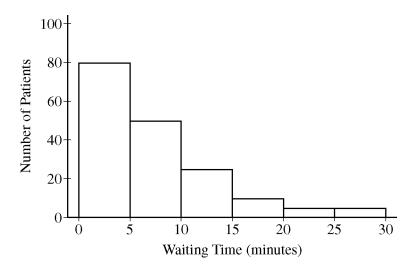
- (A) 8
- (B) 10
- (C) 16
- (D) 40
- (E) 50



- 3. The histogram above shows the number of minutes needed by 45 students to finish playing a computer game. Which of the following statements is correct?
  - (A) The distribution is skewed to the right.
  - (B) The distribution is skewed to the left.
  - (C) The distribution appears to be normal.
  - (D) The distribution appears to be chi-square.
  - (E) The distribution appears to be uniform.

- 4. A bank surveyed all of its 60 employees to determine the proportion who participate in volunteer activities. Which of the following statements is true?
  - (A) The bank should not use the data from this survey because this is an observational study.
  - (B) The bank can use the result of this survey to prove that working for the bank causes employees to participate in volunteer activities.
  - (C) The bank did not select a random sample of employees, so the survey will not provide the bank with useful information.
  - (D) The bank would have to use the survey data to construct a confidence interval in order to estimate the proportion of employees who participate in volunteer activities.
  - (E) The bank does not need to use an inference procedure to determine the proportion of employees who participate in volunteer activities because the survey was a census of all employees.

5. The histogram below displays the frequencies of waiting times, in minutes, for 175 patients in a dentist's office.



Which of the following could be the median of the waiting times, in minutes?

- (A) 2.50
- (B) 7.25
- (C) 12.25
- (D) 15.00
- (E) 17.50

6. Data were collected on the amount, in dollars, that individual customers spent on dinner in an Italian restaurant. The quartiles for these data are given below.

Q1	Q2	Q3
\$36.27	\$44.27	\$58.97

Which of the following statements must be true for these customers?

- (A) At least half of the customers spent less than or equal to \$44.27 and at least half spent greater than or equal to \$44.27.
- (B) Seventy-five percent of the customers spent between \$36.27 and \$58.97.
- (C) Twenty-five percent of the customers spent less than or equal to \$58.97 and the remaining 75 percent spent greater than or equal to \$58.97.
- (D) The mean amount spent by customers is \$44.27.
- (E) A majority of customers spent \$44.27.

- 7. The weight of adult male grizzly bears living in the wild in the continental United States is approximately normally distributed with a mean of 500 pounds and a standard deviation of 50 pounds. The weight of adult female grizzly bears is approximately normally distributed with a mean of 300 pounds and a standard deviation of 40 pounds. Approximately, what would be the weight of a female grizzly bear with the same standardized score (*z*-score) as a male grizzly bear with a weight of 530 pounds?
  - (A) 276 pounds
  - (B) 324 pounds
  - (C) 330 pounds
  - (D) 340 pounds
  - (E) 530 pounds

8. A company sells concrete in batches of 5 cubic yards. The probability distribution of *X*, the number of cubic yards sold in a single order for concrete from this company, is shown in the table below.

X = the number of cubic yards	10	15	20	25	30
Probability	0.15	0.25	0.25	0.30	0.05

The expected value of the probability distribution of X is 19.25 and the standard deviation is 5.76. There is a fixed cost to deliver the concrete. The profit Y, in dollars, for a particular order can be described by Y = 75X - 100. What is the standard deviation of Y?

- (A) \$332.00
- (B) \$432.00
- (C) \$532.00
- (D) \$1,343.75
- (E) \$1,400.00

- 9. Based on a survey of a random sample of 900 adults in the United States, a journalist reports that 60 percent of adults in the United States are in favor of increasing the minimum hourly wage. If the reported percent has a margin of error of 2.7 percentage points, which of the following is closest to the level of confidence?
  - (A) 80.0%
  - (B) 90.0%
  - (C) 95.0%
  - (D) 95.5%
  - (E) 99.0%

- 10. A compact disc (CD) manufacturer wanted to determine which of two different cover designs for a newly released CD will generate more sales. The manufacturer chose 70 stores to sell the CD. Thirty-five of these stores were randomly assigned to sell CDs with one of the cover designs and the other 35 were assigned to sell the CDs with the other cover design. The manufacturer recorded the number of CDs sold at each of the stores and found a significant difference between the mean number of CDs sold for the two cover designs. Which of the following gives the conclusion that should be made based on the results and provides the best explanation for the conclusion?
  - (A) It is <u>not</u> reasonable to conclude that the difference in sales was caused by the different cover designs because this was not an experiment.
  - (B) It is <u>not</u> reasonable to conclude that the difference in sales was caused by the different cover designs because there was no control group for comparison.
  - (C) It is <u>not</u> reasonable to conclude that the difference in sales was caused by the different cover designs because the 70 stores were not randomly chosen.
  - (D) It is reasonable to conclude that the difference in sales was caused by the different cover designs because the cover designs were randomly assigned to stores.
  - (E) It is reasonable to conclude that the difference in sales was caused by the different cover designs because the sample size was large.

- 11. The manager of a public swimming pool wants to compare the effectiveness of two laundry detergents, Detergent A and Detergent B, in cleaning the towels that are used daily. As each dirty towel is turned in, it is placed into the only washing machine on the premises. When the washing machine contains 20 towels, the manager flips a coin to determine whether Detergent A or Detergent B will be used for that load. The cleanliness of the load of towels is rated on a scale of 1 to 10 by a person who does not know which detergent was used. The manager continues this experiment for many days. Which of the following best describes the manager's study?
  - (A) A completely randomized design
  - (B) A randomized block design with Detergent A and Detergent B as blocks
  - (C) A randomized block design with the washing machine as the block
  - (D) A matched-pairs design with Detergent A and Detergent B as the pair
  - (E) An observational study

- 12. In the design of a survey, which of the following best explains how to minimize response bias?
  - (A) Increase the sample size.
  - (B) Decrease the sample size.
  - (C) Randomly select the sample.
  - (D) Increase the number of questions in the survey.
  - (E) Carefully word and field-test survey questions.

- 13. For a sample of 42 rabbits, the mean weight is 5 pounds and the standard deviation of weights is 3 pounds. Which of the following is most likely true about the weights for the rabbits in this sample?
  - (A) The distribution of weights is approximately normal because the sample size is 42, and therefore the central limit theorem applies.
  - (B) The distribution of weights is approximately normal because the standard deviation is less than the mean.
  - (C) The distribution of weights is skewed to the right because the least possible weight is within 2 standard deviations of the mean.
  - (D) The distribution of weights is skewed to the left because the least possible weight is within 2 standard deviations of the mean.
  - (E) The distribution of weights has a median that is greater than the mean.

14. When conducting a large sample test of  $H_0: p = p_0$  for a single proportion, the test statistic is

$$z = \frac{(\hat{p} - p_0)}{\sqrt{\frac{p_0(1 - p_0)}{n}}}, \text{ where } \hat{p} \text{ is the sample proportion. Which of the following best explains the justification}$$

for the denominator of this test statistic?

- (A) The standard deviation of  $\hat{p}$  is known when the null hypothesis is true.
- (B) The standard deviation of  $\hat{p}$  is known when the alternative hypothesis is true.
- (C) The sample size is large and therefore the standard deviation of  $p_0$  is approximated well.
- (D) The standard deviation of  $p_0$  is known when the null hypothesis is true.
- (E) The standard deviation of  $p_0$  is known when the alternative hypothesis is true.

- 15. A polling firm is interested in surveying a representative sample of registered voters in the United States. The firm has automated its sampling so that random phone numbers within the United States are called. Each time a number is called, the procedure below is followed.
  - If there is no response or if an answering machine is reached, another number is automatically called.
  - If a person answers, a survey worker verifies that the person is at least 18 years of age.
  - If the person is not at least 18 years of age, no response is recorded, and another number is called.
  - If the person is at least 18 years of age, that person is surveyed.

Some people claim the procedure being used does not permit the results to be extended to all registered voters. Which of the following is NOT a legitimate concern about the procedure being used?

- (A) Registered voters with children under the age of 18 years may be underrepresented in the sample.
- (B) Registered voters with unlisted telephone numbers may be underrepresented in the sample.
- (C) Registered voters who have more than one telephone number may be overrepresented in the sample.
- (D) Registered voters who live in households consisting of more than one voter may be underrepresented.
- (E) People who are not registered to vote may bias the sample results.

- 16. A complex electronic device contains three components, A, B, and C. The probabilities of failure for each component in any one year are 0.01, 0.03, and 0.04, respectively. If any one component fails, the device will fail. If the components fail independently of one another, what is the probability that the device will <u>not</u> fail in one year?
  - (A) Less than 0.01
  - (B) 0.078
  - (C) 0.080
  - (D) 0.922
  - (E) Greater than 0.99

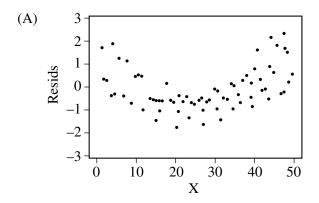
- 17. A large-sample 98 percent confidence interval for the proportion of hotel reservations that are canceled on the intended arrival day is (0.048, 0.112). What is the point estimate for the proportion of hotel reservations that are canceled on the intended arrival day from which this interval was constructed?
  - (A) 0.032
  - (B) 0.064
  - (C) 0.080
  - (D) 0.160
  - (E) It cannot be determined from the information given.

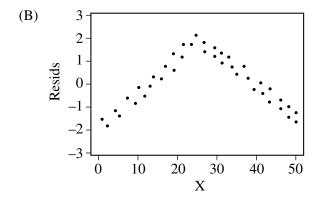
- 18. When using a one-sample *t*-procedure to construct a confidence interval for the mean of a finite population, a condition is that the population size be at least 10 times the sample size. The reason for the condition is to ensure that
  - (A) the sample size is large enough
  - (B) the central limit theorem is applicable for the sample mean
  - (C) the sample standard deviation is a good approximation of the population standard deviation
  - (D) the degree of dependence among observations is negligible
  - (E) the sampling method is not biased

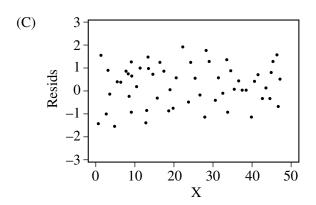
- 19. A nonprofit organization plans to hold a raffle to raise funds for its operations. A total of 1,000 raffle tickets will be sold for \$1.00 each. After all the tickets are sold, one ticket will be selected at random and its owner will receive \$50.00. The expected value for the net gain for each ticket is -\$0.95. What is the meaning of the expected value in this context?
  - (A) The ticket owners lose an average of \$0.05 per raffle ticket.
  - (B) The ticket owners lose an average of \$0.95 per raffle ticket.
  - (C) Each ticket owner will lose \$0.95 per raffle ticket.
  - (D) A ticket owner would have to purchase 19 more tickets for the expected value of his or her net gain to increase to \$0.00.
  - (E) A ticket owner has a 95 percent chance of having a ticket that is not selected.

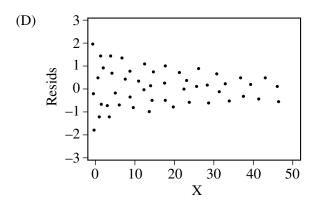
- 20. Suppose that on a hypothesis test for a single population mean,  $H_a$ :  $\mu$  < 10. Assume that  $H_a$  is true. For a fixed sample size and significance level  $\alpha$ , the power of the test will be greatest if the actual mean is which of the following?
  - (A) 8
  - (B) 9
  - (C) 10
  - (D) 11
  - (E) 13

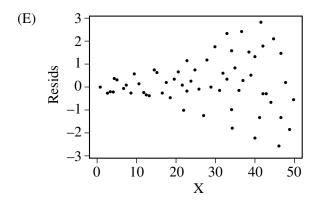
21. The residual plots from five different least squares regression lines are shown below. Which of the plots provides the strongest evidence that its regression line is an appropriate model for the data and is consistent with the assumptions required for inference for regression?











- 22. A random sample of 50 students at a large high school resulted in a 95 percent confidence interval for the mean number of hours of sleep per day of (6.73, 7.67). Which of the following statements best summarizes the meaning of this confidence interval?
  - (A) About 95% of all random samples of 50 students from this population would result in a 95% confidence interval of (6.73, 7.67).
  - (B) About 95% of all random samples of 50 students from this population would result in a 95% confidence interval that covered the population mean number of hours of sleep per day.
  - (C) 95% of the students in the survey reported sleeping between 6.73 and 7.67 hours per day.
  - (D) 95% of the students in this high school sleep between 6.73 and 7.67 hours per day.
  - (E) A student selected at random from this population sleeps between 6.73 and 7.67 hours per day for 95% of the time.

23. A local company is interested in supporting environmentally friendly initiatives such as carpooling among employees. The company surveyed all of the 200 employees at the downtown offices. Employees responded as to whether or not they own a car and to the location of the home where they live. The results are shown in the table below.

			Location of H	ome	
		Downtown Area In the City	Elsewhere In the City	Outside the City	Total
Car	Yes	10	15	35	60
Ownership	No	60	55	25	140
	Total	70	70	60	200

Which of the following statements about a randomly chosen person from these 200 employees is true?

- (A) If the person owns a car, he or she is more likely to live elsewhere in the city than to live in the downtown area in the city.
- (B) If the person does not own a car, he or she is more likely to live outside the city than to live in the city (downtown area or elsewhere).
- (C) The person is more likely to own a car if he or she lives in the city (downtown area or elsewhere) than if he or she lives outside the city.
- (D) The person is more likely to live in the downtown area in the city than elsewhere in the city.
- (E) The person is more likely to own a car than not to own a car.

24. A random sample of 432 voters revealed that 100 are in favor of a certain bond issue. A 95 percent confidence interval for the proportion of the population of voters who are in favor of the bond issue is

(A) 
$$100 \pm 1.96 \sqrt{\frac{0.5(0.5)}{432}}$$

(B) 
$$100 \pm 1.645 \sqrt{\frac{0.5(0.5)}{432}}$$

(C) 
$$100 \pm 1.96 \sqrt{\frac{0.231(0.769)}{432}}$$

(D) 
$$0.231 \pm 1.96\sqrt{\frac{0.231(0.769)}{432}}$$

(E) 
$$0.231 \pm 1.645 \sqrt{\frac{0.231(0.769)}{432}}$$

- 25. The commuting time for a student to travel from home to a college campus is normally distributed with a mean of 30 minutes and a standard deviation of 5 minutes. If the student leaves home at 8:25 A.M., what is the probability that the student will arrive at the college campus later than 9 A.M.?
  - (A) 0.16
  - (B) 0.32
  - (C) 0.50
  - (D) 0.84
  - (E) 1.00

26. In 2009 a survey of Internet usage found that 79 percent of adults age 18 years and older in the United States use the Internet. A broadband company believes that the percent is greater now than it was in 2009 and will conduct a survey. The company plans to construct a 98 percent confidence interval to estimate the current percent and wants the margin of error to be no more than 2.5 percentage points. Assuming that at least 79 percent of adults use the Internet, which of the following should be used to find the sample size (*n*) needed?

(A) 
$$1.96\sqrt{\frac{(0.5)}{n}} \le 0.025$$

(B) 
$$1.96\sqrt{\frac{(0.5)(0.5)}{n}} \le 0.025$$

(C) 
$$2.33\sqrt{\frac{(0.5)(0.5)}{n}} \le 0.05$$

(D) 
$$2.33\sqrt{\frac{(0.79)(0.21)}{n}} \le 0.025$$

(E) 
$$2.33\sqrt{\frac{(0.79)(0.21)}{n}} \le 0.05$$

27. A manufacturer claims its Brand A battery lasts longer than its competitor's Brand B battery. Nine batteries of each brand are tested independently, and the hours of battery life are shown in the table below.

Brand A	88	85	80	81	72	90	85	85	84
Brand B	80	79	77	82	75	81	77	73	78

Provided that the assumptions for inference are met, which of the following tests should be conducted to determine if Brand A batteries do, in fact, last longer than Brand B batteries?

- (A) A one-sided, paired t-test
- (B) A one-sided, two-sample *t*-test
- (C) A two-sided, two-sample *t*-test
- (D) A one-sided, two-sample z-test
- (E) A two-sided, two-sample *z*-test

- 28. An experimenter conducted a two-tailed hypothesis test on a set of data and obtained a *p*-value of 0.44. If the experimenter had conducted a one-tailed test on the same set of data, which of the following is true about the possible *p*-value(s) that the experimenter could have obtained?
  - (A) The only possible p-value is 0.22.
  - (B) The only possible p-value is 0.44.
  - (C) The only possible p-value is 0.88.
  - (D) The possible *p*-values are 0.22 and 0.78.
  - (E) The possible p-values are 0.22 and 0.88.

29. A randomized experiment was performed to determine whether two fertilizers, A and B, give different yields of tomatoes. A total of 33 tomato plants were grown; 16 using fertilizer A, and 17 using fertilizer B. The distributions of the data did not show marked skewness and there were no outliers in either data set. The results of the experiment are shown below.

	Fertilizer A	Fertilizer B
Average number of tomatoes per plant	19.54	23.39
Standard deviation	3.68	4.93
Number of plants	16	17

Which of the following statements best describes the conclusion that can be drawn from this experiment?

- (A) There is no statistical evidence of difference in the yields between fertilizer A and fertilizer B (p > 0.15).
- (B) There is a borderline statistically significant difference in the yields between fertilizer A and fertilizer B (0.10 .
- (C) There is evidence of a statistically significant difference in the yields between fertilizer A and fertilizer B (0.05 .
- (D) There is evidence of a statistically significant difference in the yields between fertilizer A and fertilizer B (0.01 .
- (E) There is evidence of a statistically significant difference in the yields between fertilizer A and fertilizer B (p < 0.01).

30. In order to plan its next advertising campaign, the Trendy Motor Vehicle Company is investigating whether the type of vehicle and the color of vehicle are related. Each person in a random sample of size 275 selected from the company's mailing list was classified according to the type (car or truck) and the color of vehicle he or she drove. The data are shown in the table below.

			Vehicle Color							
Red Black White Tan										
Vehicle	Car	35	23	41	21	12				
Type	Truck	27	55	39	12	10				

Which of the following procedures would be most appropriate to use for investigating whether there is a relationship between vehicle type and color?

- (A) A two-sample *t*-test
- (B) A two-sample z-test
- (C) A matched pairs *t*-test
- (D) A chi-square goodness-of-fit test
- (E) A chi-square test of independence

- 31. A large number of randomized experiments were conducted to determine whether taking a particular drug regularly would decrease the chance of getting a certain disease. For each of the experiments, the drug effect is the difference between the proportion of people taking the drug who got the disease and the proportion of people taking a placebo who got the disease. If the drug had no effect whatsoever, which of the following experimental results would be anticipated?
  - I. p-values will be greater than 0.05 for about 95 percent of the experiments.
  - II. There will be about an equal number of experiments showing positive and negative values of drug effect.
  - III. When 95 percent confidence intervals for the population drug effect are constructed, those confidence intervals include 0 about 95 percent of the time.
  - (A) I only
  - (B) II only
  - (C) III only
  - (D) I and II only
  - (E) I, II, and III

32. As part of a class project at a large university, Amber selected a random sample of 12 students in her major field of study. All students in the sample were asked to report their number of hours spent studying for the final exam and their score on the final exam. A regression analysis on the data produced the following partial computer output.

Predictor Constant	Coef 62.328	SE Coef 4.570	T 13.64	P 0.000
Study Hours	2.697	0.745	3.62	0.005
S = 5.505	R-sq = 56.7%			

Amber wants to compute a 95 percent confidence interval for the slope of the least squares regression line in the population of all students in her major field of study. Assuming that conditions for inference are satisfied, which of the following gives the margin of error for the confidence interval?

- (A) (2.228)(0.745)
- (B)  $(2.228)\left(\frac{0.745}{\sqrt{12}}\right)$
- (C) (2.228)(5.505)
- (D)  $(2.228) \left(\frac{5.505}{\sqrt{12}}\right)$
- (E) (2.228)(2.697)

- 33. A mathematics competition uses the following scoring procedure to discourage students from guessing (choosing an answer randomly) on the multiple-choice questions. For each correct response, the score is 7. For each question left unanswered, the score is 2. For each incorrect response, the score is 0. If there are 5 choices for each question, what is the minimum number of choices that the student must eliminate before it is advantageous to guess among the rest?
  - (A) 0
  - (B) 1
  - (C) 2
  - (D) 3
  - (E) 4

- 34. The probability that a new microwave oven will stop working in less than 2 years is 0.05. The probability that a new microwave oven is damaged during delivery and stops working in less than 2 years is 0.04. The probability that a new microwave oven is damaged during delivery is 0.10. Given that a new microwave oven is damaged during delivery, what is the probability that it stops working in less than 2 years?
  - (A) 0.05
  - (B) 0.06
  - (C) 0.10
  - (D) 0.40
  - (E) 0.50

- 35. Perchlorate is a chemical used in rocket fuel. People who live near a former rocket-testing site are concerned that perchlorate is present in unsafe amounts in their drinking water. Drinking water is considered safe when the average level of perchlorate is 24.5 parts per billion (ppb) or less. A random sample of 28 water sources in this area produces a mean perchlorate measure of 25.3 ppb. Which of the following is an appropriate alternative hypothesis that addresses their concern?
  - (A)  $H_a: \mu < 25.3$
  - (B)  $H_a: \mu > 25.3$
  - (C)  $H_a: \mu < 24.5$
  - (D)  $H_a: \mu > 24.5$
  - (E)  $H_a: \mu \neq 24.5$

36. The computer output below shows the result of a linear regression analysis for predicting the concentration of zinc, in parts per million (ppm), from the concentration of lead, in ppm, found in fish from a certain river.

Response va	riable is Zinc	(ppm)		
Variable Constant Lead (ppm)	Coefficient 16.3 19.0	Std Dev 4.90 1.89	T 3.32 10.01	P 0.003 0.000
S = 16.17	R-Sq = 82.0	%		

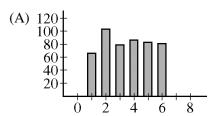
Which of the following statements is a correct interpretation of the value 19.0 in the output?

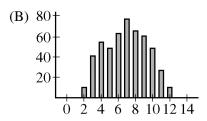
- (A) On average there is a predicted increase of 19.0 ppm in concentration of lead for every increase of 1 ppm in concentration of zinc found in the fish.
- (B) On average there is a predicted increase of 19.0 ppm in concentration of zinc for every increase of 1 ppm in concentration of lead found in the fish.
- (C) The predicted concentration of zinc is 19.0 ppm in fish with no concentration of lead.
- (D) The predicted concentration of lead is 19.0 ppm in fish with no concentration of zinc.
- (E) Approximately 19% of the variability in zinc concentration is predicted by its linear relationship with lead concentration.

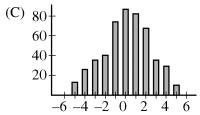
- 37. There were 5,317 previously owned homes sold in a western city in the year 2000. The distribution of the sales prices of these homes was strongly right-skewed, with a mean of \$206,274 and a standard deviation of \$37,881. If all possible simple random samples of size 100 are drawn from this population and the mean is computed for each of these samples, which of the following describes the sampling distribution of the sample mean?
  - (A) Approximately normal with mean \$206,274 and standard deviation \$3,788
  - (B) Approximately normal with mean \$206,274 and standard deviation \$37,881
  - (C) Approximately normal with mean \$206,274 and standard deviation \$520
  - (D) Strongly right-skewed with mean \$206,274 and standard deviation \$3,788
  - (E) Strongly right-skewed with mean \$206,274 and standard deviation \$37,881

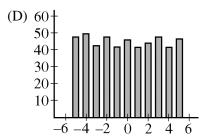
- 38. A physician believes that the exercise habits of East Coast adults are different from the exercise habits of West Coast adults. To study this, she gathers information on the number of hours of exercise per week from a random sample of East Coast adults and a random sample of West Coast adults. Which of the following might be an appropriate null hypothesis for this study?
  - (A) The average number of hours of exercise per week for East Coast adults is different from the average number of hours of exercise per week for West Coast adults.
  - (B) The average number of hours of exercise per week for East Coast adults is the same as the average number of hours of exercise per week for West Coast adults.
  - (C) The average number of hours of exercise per week for East Coast adults is greater than the average number of hours of exercise per week for West Coast adults.
  - (D) The average number of hours of exercise per week for East Coast adults is less than the average number of hours of exercise per week for West Coast adults.
  - (E) The probability is 0.5 that an East Coast adult and a West Coast adult exercise an equal number of hours per week.

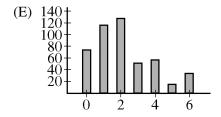
39. For a roll of a fair die, each of the outcomes 1, 2, 3, 4, 5, or 6 is equally likely. A red die and a green die are rolled simultaneously, and the difference of the outcomes (red – green) is computed. This is repeated for a total of 500 rolls of the pair of dice. Which of the following graphs best represents the most reasonable distribution of the differences?











40. The probability of winning a certain game is 0.5. If at least 70 percent of the games in a series of n games are won, the player wins a prize. If the possible choices for n are

n=10, n=20, and n=100,

which value of n should the player choose in order to maximize the probability of winning a prize?

- (A) n=10 only
- (B) n=20 only
- (C) n=100 only
- (D) n=10 or n=20 only; the probabilities are the same.
- (E) n=10 or n=20 or n=100; the probabilities are the same.

### **END OF SECTION I**

# IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET
- WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET
- TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET

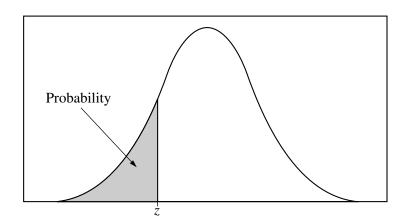


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

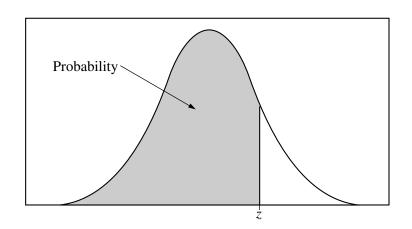


Table entry for z is the probability lying below z.

Table A (Continued)

	(									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

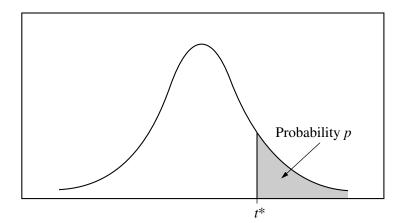


Table entry for p and C is the point  $t^*$  with probability p lying above it and probability C lying between  $-t^*$  and  $t^*$ .

Table B t distribution critical values

						Tail prol	bability <i>p</i>					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

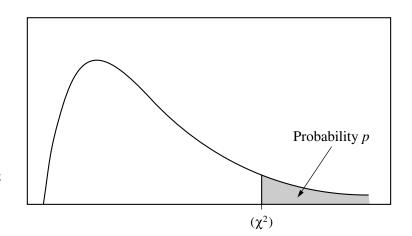


Table entry for p is the point  $(\chi^2)$  with probability p lying above it.

Table C  $\chi^2$  critical values

						Tail prob	pability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

# Section II: Free-Response Questions

This is the free-response section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

# AP® Statistics Exam

**SECTION II: Free Response** 

2012

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

# At a Glance

### **Total Time**

1 hour, 30 minutes **Number of Questions** 

6

Percent of Total Score

**Writing Instrument** 

Either pencil or pen with black or dark blue ink

**Electronic Device** 

Graphing calculator expected

### Part A

Number of Questions

**Suggested Time** 

1 hour, 5 minutes

Percent of Section II Score

### Part B

Number of Questions

Suggested Time 25 minutes

Percent of Section II Score

IMPO	RTANT I	ldentificat	ion Ini	ormati	on

### PLEASE PRINT WITH PEN:

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  First letter of your first name
- 2. Date of birth

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Six-digit school code

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4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights.

### **Instructions**

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.



Formulas begin on page 3. Questions begin on page 6. Tables begin on page 18.

# **Formulas**

(I) Descriptive Statistics

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \overline{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \overline{x}}{s_x} \right) \left( \frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \overline{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_X = \sum x_i p_i$$

$$Var(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu_{\chi} = np$$

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size n from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\overline{X}} = \mu$$

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

# (III) Inferential Statistics

 $Standardized \ test \ statistic: \frac{statistic - parameter}{standard \ deviation \ of \ statistic}$ 

Confidence interval: statistic ± (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic		
Sample Mean	$\frac{\sigma}{\sqrt{n}}$		
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$		

Two-Sample

Statistic	Standard Deviation of Statistic	
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	
	Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} + \frac{p_2(1-p_2)}{n_2}$ Special case when $p_1 = p_2$	
	$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	

Chi-square test statistic = 
$$\sum \frac{(observed - expected)^2}{expected}$$

# STATISTICS SECTION II

### Part A

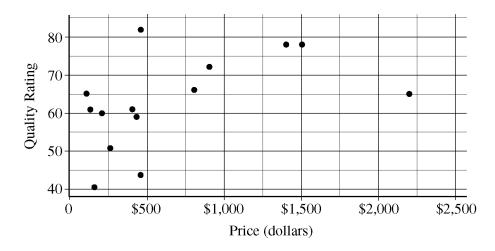
# **Questions 1-5**

# Spend about 65 minutes on this part of the exam.

# Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

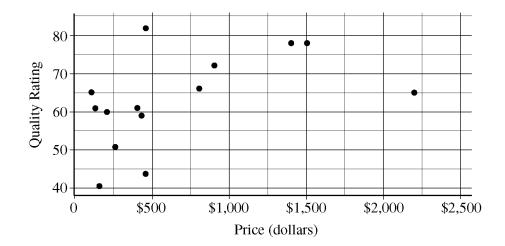
1. The scatterplot below displays the price in dollars and quality rating for 14 different sewing machines.



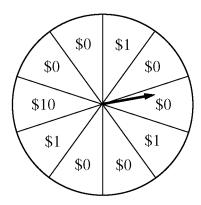
(a) Describe the nature of the association between price and quality rating for the sewing machines.

(b) One of the 14 sewing machines substantially affects the appropriateness of using a linear regression model to predict quality rating based on price. Report the approximate price and quality rating of that machine and explain your choice.

(c) Chris is interested in buying one of the 14 sewing machines. He will consider buying only those machines for which there is no other machine that has both higher quality and lower price. On the scatterplot reproduced below, circle all data points corresponding to machines that Chris will consider buying.



2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of \$2 is required to play the game. For each \$2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

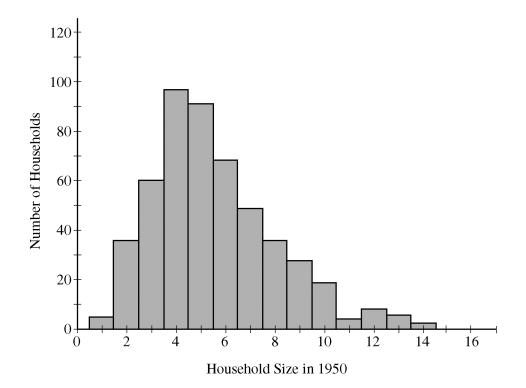
x	\$2	\$1	-\$8
P(x)			

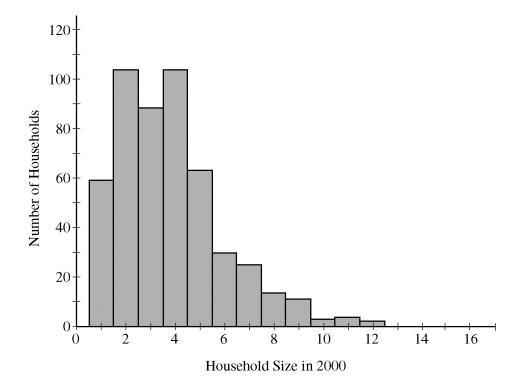
(b) What is the expected value of the net contribution to the charity for one play of the game?

(c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

3. Independent random samples of 500 households were taken from a large metropolitan area in the United States for the years 1950 and 2000. Histograms of household size (number of people in a household) for the years are shown below.





(a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000.	
(b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using two-sample <i>t</i> -procedure, and explain whether the conditions for inference are met.	g a
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4. A survey organization conducted telephone interviews in December 2008 in which 1,009 randomly selected adults in the United States responded to the following question.

At the present time, do you think television commercials are an effective way to promote a new product?

Of the 1,009 adults surveyed, 676 responded "yes." In December 2007, 622 of 1,020 randomly selected adults in the United States had responded "yes" to the same question. Do the data provide convincing evidence that the proportion of adults in the United States who would respond "yes" to the question changed from December 2007 to December 2008?

5. A recent report stated that less than 35 percent of the adult residents in a certain city will be able to pass a physical fitness test. Consequently, the city's Recreation Department is trying to convince the City Council to fund more physical fitness programs. The council is facing budget constraints and is skeptical of the report. The council will fund more physical fitness programs only if the Recreation Department can provide convincing evidence that the report is true.

The Recreation Department plans to collect data from a sample of 185 adult residents in the city. A test of significance will be conducted at a significance level of  $\alpha = 0.05$  for the following hypotheses.

$$H_0: p = 0.35$$

$$H_a: p < 0.35,$$

where p is the proportion of adult residents in the city who are able to pass the physical fitness test.

(a) Describe what a Type II error would be in the context of the study, and also describe a consequence of making this type of error.

(b) The Recreation Department recruits 185 adult residents who volunteer to take the physical fitness test. The test is passed by 77 of the 185 volunteers, resulting in a *p*-value of 0.97 for the hypotheses stated above. If it was reasonable to conduct a test of significance for the hypotheses stated above using the data collected from the 185 volunteers, what would the *p*-value of 0.97 lead you to conclude?

(c) Describe the primary flaw in the study described in part (b), and explain why it is a concern.

# STATISTICS SECTION II

# Part B

# **Question 6**

# Spend about 25 minutes on this part of the exam. Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- 6. Two students at a large high school, Peter and Rania, wanted to estimate  $\mu$ , the mean number of soft drinks that a student at their school consumes in a week. A complete roster of the names and genders for the 2,000 students at their school was available. Peter selected a simple random sample of 100 students. Rania, knowing that 60 percent of the students at the school are female, selected a simple random sample of 60 females and an independent simple random sample of 40 males. Both asked all of the students in their samples how many soft drinks they typically consume in a week.
  - (a) Describe a method Peter could have used to select a simple random sample of 100 students from the school.

Peter and Rania conducted their studies as described. Peter used the sample mean  $\overline{X}$  as a point estimator for  $\mu$ . Rania used  $\overline{X}_{\text{overall}} = (0.6)\overline{X}_{\text{female}} + (0.4)\overline{X}_{\text{male}}$  as a point estimator for  $\mu$ , where  $\overline{X}_{\text{female}}$  is the mean of the sample of 60 females and  $\overline{X}_{\text{male}}$  is the mean of the sample of 40 males.

Summary statistics for Peter's data are shown in the table below.

Variable	N	Mean	Standard Deviation
Number of soft drinks	100	5.32	4.13

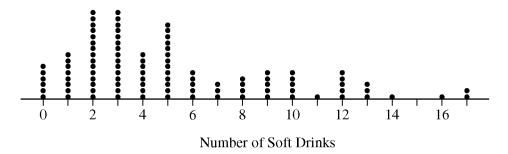
(b) Based on the summary statistics, calculate the estimated standard deviation of the sampling distribution (sometimes called the standard error) of Peter's point estimator  $\overline{X}$ .

Summary statistics for Rania's data are shown in the table below.

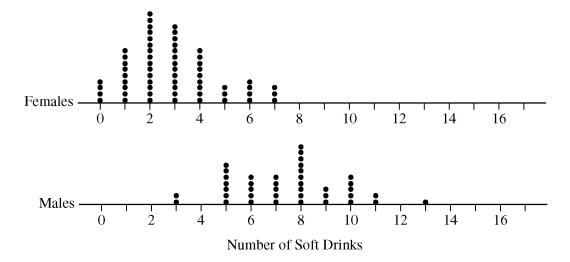
Variable	Gender	N	Mean	Standard Deviation
Number of	Female	60	2.90	1.80
soft drinks	Male	40	7.45	2.22

(c) Based on the summary statistics, calculate the estimated standard deviation of the sampling distribution of Rania's point estimator  $\overline{X}_{\text{overall}} = (0.6)\overline{X}_{\text{female}} + (0.4)\overline{X}_{\text{male}}$ .

A dotplot of Peter's sample data is given below.



Comparative dotplots of Rania's sample data are given below.



(d) Using the dotplots above, explain why Rania's point estimator has a smaller estimated standard deviation than the estimated standard deviation of Peter's point estimator.

# STOP

# **END OF EXAM**

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX(ES) ON THE COVER(S).
- MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMS YOU HAVE TAKEN THIS YEAR.

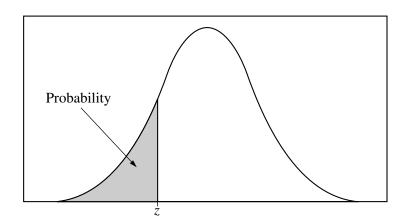


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

1				-						
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

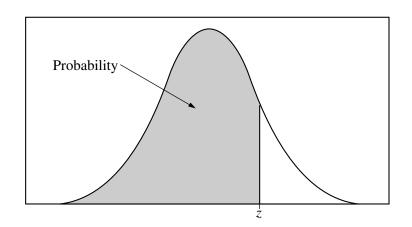


Table entry for z is the probability lying below z.

Table A (Continued)

Tubic 11	(Commi	iica)								
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

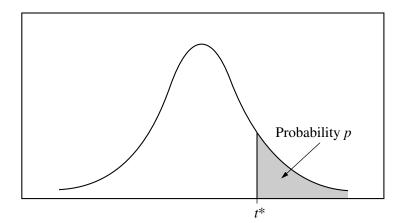


Table entry for p and C is the point  $t^*$  with probability p lying above it and probability C lying between  $-t^*$  and  $t^*$ .

Table B t distribution critical values

able B	l distri	oution cri	icai vaiues									
						Tail prol	bability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

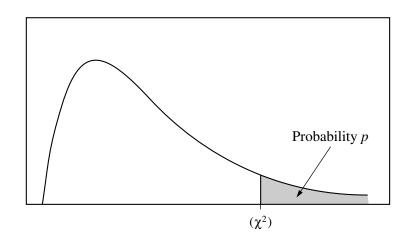


Table entry for p is the point  $(\chi^2)$  with probability p lying above it.

Table C  $\chi^2$  critical values

						Tail prob	ability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

**Multiple-Choice Answer Key** 

The following contains the answers to the multiple-choice questions in this exam.

# Answer Key for AP Statistics Practice Exam, Section I

Multiple-Choice Questions					
Question #	Key				
1	В				
2	С				
3	В				
4	Е				
5	В				
6	A				
7	В				
8	В				
9	В				
10	D				
11	A				
12	Е				
13	С				
14*	_				
15	В				
16	D				
17	С				
18	D				
19	В				

20	A
21	С
22	В
23	A
24	D
25	A
26	D
27	В
28	D
29	D
30	Е
31	Е
32	A
33	С
34	D
35	D
36	В
37	A
38	В
39	С
40	A

<sup>\*</sup>Item 14 was not used in scoring.

# Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

### Question 1

# **Intent of Question**

The primary goals of this question were to assess students' ability to (1) describe a nonlinear association based on a scatterplot; (2) describe how an unusual observation may affect the appropriateness of using a linear model for bivariate numeric data; (3) implement a decision-making criterion on data presented in a scatterplot.

# Solution

# Part (a):

The data show a weak but positive association between price and quality rating for these sewing machines. The form of the association does not appear to be linear. Among machines that cost less than \$500, there appears to be very little association between price and quality rating. But the machines that cost more than \$500 do generally have better quality ratings than those that cost less than \$500, which causes the overall association to be positive.

# Part (b):

The sewing machine that most affects the appropriateness of using a linear regression model is the one that costs about \$2,200 and has a quality rating of about 65. Although the other four sewing machines costing more than \$500 generally have higher quality ratings than those costing under \$500, their prices and quality ratings follow a trend that suggests that quality ratings may not continue to increase with higher prices, but instead may approach a maximum possible quality rating. The \$2,200 sewing machine is the most expensive of all but has a relatively low quality rating, which is consistent with a nonlinear model that approaches a maximum possible quality rating and then perhaps decreases. If a linear model were fit to all of the data, this one machine would substantially pull the regression line toward it, resulting in a poor overall fit of the line to the data.

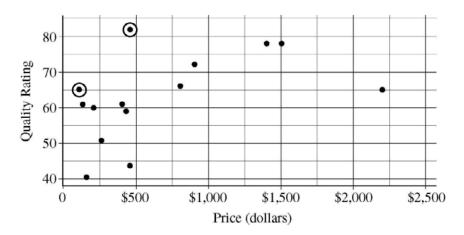
# Question 1 (continued)

# Part (c):

According to Chris's criterion, there are two sewing machine models that he will consider buying:

- 1. The model that costs a bit more than \$100 and has a quality rating of 65.
- 2. The model that costs a bit below \$500 and has a quality rating of 81 or 82.

The data points corresponding to these two machines have been circled on the scatterplot below.



# Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

# Part (a) is scored as follows:

Essentially correct (E) if the response correctly describes three aspects of association: direction (positive), strength (weak or moderate), and form (curved or nonlinear), *AND* describes the association in context.

Partially correct (P) if the response correctly describes two aspects of association in context OR

if the response describes all three aspects of association without context.

Incorrect (I) if the response fails to meet the criteria for E or P.

# Part (b) is scored as follows:

Essentially correct (E) if the response identifies the correct point with reasonable approximations to the price and quality values AND gives either of the following two explanations:

- 1. The point in conjunction with the entire collection of points appears to have a curved (or nonlinear) form.
- 2. A linear model that includes all the points would result in a poor overall fit to the data, largely owing to the presence and influence of the identified point.

# Question 1 (continued)

Partially correct (P) if the response identifies the correct point with reasonable approximations to the price and quality values *AND* gives a weak explanation of why the point affects the reasonableness of a linear model. The following are examples of weak explanations.

- 1. The point is an outlier.
- 2. Removal of the point makes the pattern more linear.
- 3. The point does not follow the linear pattern of the others.
- 4. A sewing machine this expensive should have a higher quality rating.
- 5. There is a much cheaper sewing machine with the same quality rating as this one.
- 6. The point has considerable influence on the parameters of the least squares regression line.

Incorrect if the response fails to meet the criteria for E or P.

# **Part (c)** is scored as follows:

Essentially correct (E) if the correct two points are circled AND no other points are circled.

Partially correct (P) if the correct two points are circled *AND* one or two other points are circled. *OR* 

if only one of the two correct points is circled AND at most one other point is circled.

Incorrect (I) if the response fails to meet the criteria for E or P.

# 4 Complete Response

All three parts essentially correct

# 3 Substantial Response

Two parts essentially correct and one part partially correct

# 2 Developing Response

OR
One part essentially correct and one part incorrect
OR
One part essentially correct and two parts partially correct
OR
One part essentially correct and one part partially correct
(BUT see the exception noted with an asterisk below)
OR
All three parts partially correct

# 1 Minimal Response

One part essentially correct and two parts incorrect

OR

\*Part (c) essentially correct, part (b) partially correct, and part (a) incorrect

OR

Two parts partially correct and one part incorrect

### Question 2

# **Intent of Question**

The primary goals of this question were to assess students' ability to (1) perform calculations and compute expected values related to a discrete probability distribution; (2) implement a normal approximation based on the central limit theorem.

# **Solution**

# Part (a):

By counting the number of sectors for each value and dividing by 10, the probability distribution is calculated to be:

X	\$2	\$1	-\$8
P(x)	0.6	0.3	0.1

# Part (b):

The expected value of the net contribution for one play of the game is:

$$E(x) = \$2(0.6) + \$1(0.3) + (-\$8)(0.1) = \$0.70$$
 (or 70 cents).

# Part (c):

The expected contribution after n plays is \$0.70n. Setting this to be at least \$500 and solving for n gives:

$$0.70n \ge 500$$
, so  $n \ge \frac{500}{0.70} \approx 714.286$ ,

so 715 plays are needed for the expected contribution to be at least \$500.

# Part (d):

The normal approximation is appropriate because the very large sample size (n = 1,000) ensures that the central limit theorem holds. Therefore, the sample mean of the contributions from 1,000 plays has an approximately normal distribution, and so the sum of the contributions from 1,000 plays also has an approximately normal distribution.

The z-score is 
$$\frac{500 - 700}{92.79} \approx -2.155$$
.

The probability that a standard normal random variable exceeds this z-score of -2.155 is 0.9844. Therefore, the charity can be very confident about gaining a net contribution of at least \$500 from 1,000 plays of the game.

# Question 2 (continued)

# Scoring

This question is scored in three sections. Section 1 consists of parts (a) and (b); section 2 consists of part (c); and section 3 consists of part (d). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

# **Section 1** is scored as follows:

Essentially correct (E) if all three probabilities are filled in correctly in the table in part (a) *AND* the expected value is calculated correctly in part (b), with work shown.

Partially correct (P) if all three probabilities are filled in correctly in the table in part (a) AND the expected value is not calculated correctly in part (b),

OR

the probabilities in part (a) are not all correct *AND* the expected value in part (b) is calculated appropriately from the probabilities given in part (a) or from the correct probabilities.

Incorrect (I) if the response does not meet the criteria for E or P.

### **Section 2** is scored as follows:

Essentially correct (E) if the response addresses the following two components:

- 1. Provides a solution based on a reasonable calculation, equation, or inequality from the answer given in part (b).
- 2. Clearly selects the next higher integer as the answer.

Partially correct (P) if the response correctly completes component (1) listed above but not component (2).

Incorrect (I) if the response does not meet the criteria for E or P.

### **Section 3** is scored as follows:

Essentially correct (E) if the response correctly addresses the following three components:

- 1. Indicates the use of a normal distribution with the correct mean and standard deviation.
- 2. Uses the correct boundary and indicates the correct direction.
- 3. Has the correct normal probability consistent with components (1) and (2).

Partially correct (P) if the response correctly addresses exactly two of the three components listed above

Incorrect (I) if the response does not meet the criteria for E or P.

# Question 2 (continued)

### Notes

- Because the question asks students to use a normal distribution and specifies the parameter values, the response does not have to justify the normal approximation or show how to calculate the parameter values.
- If the response earns credit for component (1) but no direction has been provided for component (2), then the response earns credit for component (3) if the correct probability of 0.9844 is reported.
- If the response does not earn credit for component (1) owing to incorrect identification of the mean and/or standard deviation, then the response can still earn credit for component (2) if the boundary is calculated correctly from the mean and standard deviation indicated in component (1).

# 4 Complete Response

All three sections essentially correct

# 3 Substantial Response

Two sections essentially correct and one section partially correct

# 2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

# 1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

### Question 3

# **Intent of Question**

The primary goals of this question were to assess students' ability to (1) compare two distributions presented with histograms; (2) comment on the appropriateness of using a two-sample t-procedure in a given setting.

# Solution

# Part (a):

Household size tended to be larger in 1950 than in 2000. The histograms reveal a much larger proportion of small (1-, 2-, and 3-person) households in 2000 than in 1950. Similarly, the histograms reveal a much smaller proportion of large (5-person and larger) households in 2000 than in 1950. Also, the median household sizes can be calculated to be 5 people per household in 1950 compared with 3 or 4 people per household in 2000. The year 1950 displayed slightly more variability in household sizes than the year 2000. Although the interquartile ranges for both years are the same (3 people), the standard deviation (1950: about 2.6 people; 2000: about 2.1 people) and the range (1950: 13 people; 2000: 11 people) are larger for 1950 than for 2000. Both distributions of household size are skewed to the right. In both years, there are a few households with very large families, as large as 14 people in 1950 and 12 people in 2000.

# Part (b):

The conditions for applying a two-sample *t*-procedure are:

- 1. The data come from independent random samples or from random assignment to two groups;
- 2. The populations are normally distributed, or both sample sizes are large;
- 3. The population sizes are at least 10 (or 20) times the sample sizes.

The first condition is satisfied because independent random samples were selected for the years 1950 and 2000. The second condition is satisfied because the sample sizes (500 in each group) are quite large, despite the right skewness of the distributions of household sizes in the sample data. The third condition is satisfied because the number of households in the large metropolitan area in both 1950 and 2000 would easily exceed  $10 \times 500 = 5,000$ .

# Scoring

This question is scored in four sections. Part (a) has three components: (1) comparing the centers of the two distributions; (2) comparing variability for the two distributions; (3) identifying the shapes of both distributions and including context related to the variable of interest. Section 1 consists of part (a), component 1; section 2 consists of part (a), component 2; section 3 consists of part (a), component 3. Section 4 consists of part (b). Sections 1 and 2 are scored as essentially correct (E) or incorrect (I). Sections 3 and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

### **Section 1** is scored as follows:

Essentially correct (E) if the response correctly compares center (or location) for both distributions.

Incorrect (I) otherwise.

# Question 3 (continued)

# Section 2 is scored as follows:

Essentially correct (E) if the response correctly compares variability for both distributions.

Incorrect (I) otherwise.

# **Section 3** is scored as follows:

Essentially correct (E) if the response includes context related to the variable of interest (household size) *AND* the response correctly identifies the shapes of both distributions.

Partially correct (P) if the response correctly identifies the shapes of both distributions *BUT* does *NOT* include context related to the variable of interest (household size),

OR

if the response correctly identifies the shape of only one distribution *AND* includes context related to the variable of interest (household size).

Incorrect (I) otherwise.

### **Section 4** is scored as follows:

Essentially correct (E) if the response correctly states and checks the following two conditions.

- 1. The data come from independent random samples
- 2. Normality/sample size conditions.

Partially correct (P) if the response correctly states and checks only one of the two conditions listed above,

OR

if the response correctly refers to random samples and large sample size, *BUT* does *NOT* state and check either condition correctly.

Incorrect (I) otherwise.

Note: The population size condition does not need to be checked to earn E or P.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

- 4 Complete Response
- 3 Substantial Response
- 2 Developing Response
- 1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

### Question 4

# **Intent of Question**

The primary goal of this question was to assess students' ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess students' ability to (1) state appropriate hypotheses; (2) identify the name of an appropriate statistical test and check appropriate assumptions/conditions; (3) calculate the appropriate test statistic and p-value; (4) draw an appropriate conclusion, with justification, in the context of the study.

# **Solution**

Step 1: States a correct pair of hypotheses.

Let  $p_{07}$  represent the population proportion of adults in the United States who would have answered "yes" about the effectiveness of television commercials in December 2007. Let  $p_{08}$  represent the analogous population proportion in December 2008.

The hypotheses to be tested are  $H_0$ :  $p_{07} = p_{08}$  versus  $H_a$ :  $p_{07} \neq p_{08}$ .

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a two-sample *z*-test for comparing proportions.

Because these are sample surveys, the first condition is that the data were gathered from independent random samples from the two populations. This condition is met because we are told that the subjects were randomly selected in the two different years. Although we are not told whether the samples were selected independently, this is a reasonable assumption given that they are samples of different sizes selected in different years.

The second condition is that the sample sizes are large, relative to the proportions involved. This condition is satisfied because all sample counts (622 "yes" in 2007; 1,020 - 622 = 398 "no" in 2007; 676 "yes" in 2008; 1,009 - 676 = 333 "no" in 2008) are all at least 10 (or, are all at least 5).

An additional condition may be checked: The population sizes (more than 200 million adults in the United States) are much larger than 10 (or, 20) times the sample sizes.

Step 3: Correct mechanics, including the value of the test statistic and p-value (or rejection region).

The sample proportions who answered "yes" are:

$$\hat{p}_{07} = \frac{622}{1.020} \approx 0.6098$$
 and  $\hat{p}_{08} = \frac{676}{1.009} \approx 0.6700$ .

The combined proportion,  $\hat{p}_c$ , who answered "yes" in these two years is:

$$\hat{p}_c = \frac{622 + 676}{1,020 + 1,009} = \frac{1,298}{2,029} \approx 0.6397.$$

# Question 4 (continued)

The test statistic is:

$$z = \frac{\hat{p}_{07} - \hat{p}_{08}}{\sqrt{\hat{p}_c \left(1 - \hat{p}_c\right) \left(\frac{1}{n_{07}} + \frac{1}{n_{08}}\right)}} = \frac{0.6098 - 0.6700}{\sqrt{(0.6397)(1 - 0.6397) \left(\frac{1}{1,020} + \frac{1}{1,009}\right)}} \approx -2.82.$$

The *p*-value is  $2P(Z \le -2.82) \approx 0.0048$ .

Step 4: State a correct conclusion in the context of the study, using the result of the statistical test.

Because this p-value is smaller than any common significance level such as  $\alpha = 0.05$  or  $\alpha = 0.01$  (or, because this p-value is so small), we reject  $H_0$  and conclude that the data provide convincing (or, statistically significant) evidence that the proportion of all adults in the United States who would answer "yes" to the question about the effectiveness of television commercials changed from December 2007 to December 2008.

# Scoring

Each of steps 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

# Step 1 is scored as follows:

Essentially correct (E) if the response identifies correct parameters *AND* both hypotheses are labeled and state the correct relationship between the parameters.

Partially correct (P) if the response identifies correct parameters *OR* states correct relationships, but not both.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Either defining the parameter symbols in context, or simply using common parameter notation, such as  $p_{07}$  and  $p_{08}$ , with subscripts clearly relevant to the context, is sufficient.

# Step 2 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:

- 1. Identifies the correct test procedure (by name or by formula).
- 2. Checks for randomness.
- 3. Checks for normality.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response correctly includes one or none of the three components listed above.

# Question 4 (continued)

# **Step 3** is scored as follows:

Essentially correct (E) if the response correctly calculates both the test statistic and a p-value that is consistent with the stated alternative hypothesis.

Partially correct (P) if the response correctly calculates the test statistic but not the p-value, OR

if the response calculates the test statistic incorrectly but then calculates the correct p-value for the computed test statistic.

Incorrect (I) if the response fails to meet the criteria for E or P.

# **Step 4** is scored as follows:

Essentially correct (E) if the response provides a correct decision in context, also providing justification based on linkage between the *p*-value and conclusion.

Partially correct (P) if the response provides a correct decision, with linkage to the p-value, but not in context.

OR

if the response provides a correct decision in context, but without justification based on linkage to the *p*-value.

Incorrect (I) if the response does not meet the criteria for E or P.

*Note*: If the decision is consistent with an incorrect *p*-value from step 3, and also in context with justification based on linkage to the *p*-value, then step 4 is scored as E.

Each essentially correct (E) step counts as 1 point. Each partially correct (P) step counts as ½ point.

- 4 Complete Response
- 3 Substantial Response
- 2 Developing Response
- 1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

### Question 5

# **Intent of Question**

The primary goals of this question were to assess students' ability to (1) describe a Type II error and its consequence in a particular study; (2) draw an appropriate conclusion from a p-value; (3) describe a flaw in a study and its effect on inference from a sample to a population.

# **Solution**

# Part (a):

In the context of the study, a Type II error means failing to reject the null hypothesis that 35 percent of adult residents in the city are able to pass the test when, in reality, less than 35 percent are able to pass the test. The consequence of this error is that the council would not fund the program, and the city would continue to have a smaller proportion of physically fit residents than the council would like.

# Part (b):

Because the p-value of 0.97 is larger than  $\alpha=0.05$ , we fail to reject the null hypothesis. There is not convincing evidence that the proportion of adult residents in the city who are able to pass the physical fitness test is less than 0.35. After all, the sample proportion of  $\hat{p}=0.416$  is actually higher than 0.35, which is in the opposite direction of the alternative hypothesis.

# Part (c):

This is not a randomly selected sample because the sample was selected by recruiting volunteers. It seems reasonable to think that volunteers would be more physically fit than the population of city adults as a whole. Therefore, the sample proportion will likely overestimate the population proportion of adult residents in the city who are able to pass the physical fitness test.

# Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

# Part (a) is scored as follows:

Essentially correct (E) if the response correctly completes the following two components:

- 1. Describes the error in context by referring to the proportion of adult residents in the city who are able to pass the physical fitness test.
- 2. Describes the consequence as not funding the program and/or continuing poor physical fitness of the adult residents in the city.

# Question 5 (continued)

### Notes

- If a response provides more than one description of a Type II error, score the weakest attempt.
- Referring to the symbolic hypotheses is *not sufficient* for context.
- Referring to funding and/or the city council is *not sufficient* for context.
- If a response describes a Type II error incorrectly, the response can get the consequence component correct if it is consistent with the incorrectly described error.
- If a response provides more than one description of a Type II error, the response can get the consequence component correct if the consequence is clearly linked to one of the error descriptions and is consistent with the error to which it is linked.
- If a response gives an incomplete description of a Type II error (for example, "we fail to reject the null hypothesis that the proportion of adult residents who are able to pass is 0.35"), the response can get the consequence component correct if the consequence is consistent with the partial description of the error.
- If a response provides no description of a Type II error, the response cannot get the consequence component correct.

Partially correct (P) if the response correctly completes only one of the two components listed above.

Incorrect (I) if the response correctly completes neither of the two components listed above.

*Note*: Describing the Type II error only in terms of the consequence (for example, "They don't fund the program when they should") should get credit for the consequence but should not get credit for the error, because there is no reference to the proportion of adult residents in the city who are able to pass the test.

# **Part (b)** is scored as follows:

Essentially correct (E) if the response correctly completes the following three components:

- 1. Links the p-value to the conclusion by stating that the p-value is greater than  $\alpha=0.05$  , OR
  - by stating that the p-value is large,
  - OR
  - by correctly interpreting the p-value.
- 2. Uses context by referring to the proportion of adult residents who are able to pass the test, *OR* 
  - by referring to the funding of the program.
- 3. Makes a correct conclusion that describes the lack of evidence for the alternative hypothesis  $(H_a: p < 0.35)$ .

### Notes

- If a response includes an incorrect interpretation of the p-value, then the response cannot earn credit for the linkage component, even if the response explicitly compares the p-value to  $\alpha$  or describes the p-value as large.
- Referring to the symbolic hypotheses is not sufficient for context.
- Accepting the null hypothesis or some equivalent statement such as "the population proportion is (or is likely to be, or is about) 0.35" cannot receive credit for the conclusion component, even if the student makes additional correct statements about the alternative hypothesis.

# Question 5 (continued)

- Stating that the null hypothesis should not be rejected is not sufficient for the conclusion, because it does not address the direction of H<sub>a</sub>.
- Correctly addressing the consequence ("They don't fund the program") is sufficient if the response also indicates that the null hypothesis is not being rejected.
- Drawing a conclusion about the sample proportion (for example, "proportion who passed the test") is not sufficient for the conclusion, because it does not properly address the parameter in H<sub>a</sub>.

Partially correct (P) if the response correctly completes two of the three components listed above.

Incorrect (I) if the response correctly completes one or none of the three components listed above.

### Notes

- A response that says the p-value is very large, recognizes that the sample proportion  $(\hat{p} = 0.416)$  is greater than p = 0.35, and consequently concludes there is no evidence to support  $H_a$  (in context) is scored as essentially correct (E).
- A response that *rejects* H<sub>0</sub> is scored as incorrect (I).

# Part (c) is scored as follows:

Essentially correct (E) if the response correctly completes the following three components:

- 1. States that the sample is not random and/or says that volunteers were used.
- 2. Describes *how* the sample is "different" with regard to physical fitness or another variable related to the ability to pass the physical fitness test.
- 3. Addresses the idea of making an inference from the sample to the population by stating that the sample statistic will overestimate the population parameter or that the sample will not be representative of the population.

### Notes

- If for the first component a student provides additional proposed flaws (for example, "the sample size is too small"), score the weakest attempt.
- Saying only that the sample is different or not representative does not address *how* the sample is different.
- Saying "physically fit people will be overrepresented" or "the results cannot be generalized" or "the results will be inaccurate" lack a specific reference to the population and is not sufficient for the third component.
- Referring to "bias" is not sufficient for the first component unless the concept of bias is clearly explained (for example, saying that the sample proportion will tend to overestimate the population proportion).
- Incorrect application of statistical concepts (for example, saying that the statistic is "skewed," discussing cause and effect) results in a loss of credit for the third component.

Partially correct (P) if the response correctly addresses two of the three components listed above.

Incorrect (I) if the response correctly addresses one or none of the three components listed above.

# Question 5 (continued)

# 4 Complete Response

All three parts essentially correct

# 3 Substantial Response

Two parts essentially correct and one part partially correct

# 2 Developing Response

Two parts essentially correct and one part incorrect OROne part essentially correct and one or two parts partially correct ORThree parts partially correct

# 1 Minimal Response

OR

One part essentially correct and two parts incorrect

Two parts partially correct and one part incorrect

### Question 6

# **Intent of Question**

The primary goals of this question were to assess students' ability to (1) implement simple random sampling; (2) calculate an estimated standard deviation for a sample mean; (3) use properties of variances to determine the estimated standard deviation for an estimator; (4) explain why stratification reduces a standard error in a particular study.

# Solution

# Part (a):

Peter can number the students from 1 to 2,000 and then use a calculator or computer to generate 100 unique random numbers between 1 and 2,000 without replacement. If non-unique numbers are generated, the repeated numbers are ignored until 100 unique numbers are obtained. The students whose numbers correspond to the randomly generated numbers are then selected for the sample.

# Part (b):

The estimated standard deviation of the sampling distribution of the sample mean is:

$$\frac{S}{\sqrt{n}}$$
, or  $\frac{4.13}{\sqrt{100}} = 0.413$ .

# Part (c):

The variance of Rania's estimator is  $(0.6)^2 \text{Var}(\bar{X}_f) + (0.4)^2 \text{Var}(\bar{X}_m)$ , where  $\text{Var}(\bar{X}_f) = \frac{\sigma_f^2}{n_f}$  represents the variance of the point estimator for females and  $\text{Var}(\bar{X}_m) = \frac{\sigma_m^2}{n_m}$  represents the variance of the point estimator for males.

The estimated standard deviation is the square root of the variance. Using the respective sample standard deviations  $s_f$  and  $s_m$  for the population parameters, Rania's estimate is calculated as:

$$\sqrt{(0.6)^2 \frac{s_f^2}{n_f} + (0.4)^2 \frac{s_m^2}{n_m}} = \sqrt{(0.6)^2 \frac{(1.80)^2}{60} + (0.4)^2 \frac{(2.22)^2}{40}} \approx \sqrt{0.01944 + 0.01972} \approx 0.198.$$

# Part (d):

The comparative dotplots from Rania's data reveal that the distribution of the number of soft drinks for females appears to be quite different from that of males. In particular, the centers of the distributions appear to be significantly different. Additionally, the variability of values around the center *within* gender in each of Rania's dotplots appears to be considerably less than the variability displayed in the dotplot of Peter's data. Rania's estimator takes advantage of the decreased variability within gender because her data were obtained by sampling the two genders separately. Peter's estimator has more variability because his data were obtained from a simple random sample of all the high school students.

# Question 6 (continued)

# Scoring

Parts (a), (b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

# Part (a) is scored as follows:

Essentially correct (E) if the response provides a correct sampling procedure to obtain a simple random sample that includes sufficient information such that two knowledgeable statistics users would implement equivalent methods.

Partially correct (P) if the response provides a plausible sampling procedure for obtaining a simple random sample but does *NOT* include sufficient information as described for E.

Incorrect (I) if the response does not meet the criteria for E or P.

### Notes

- If computer-generated random numbers are used, an explicit statement about ignoring repeats is needed for sufficient information. (Sampling until 100 students are obtained conveys the idea of ignoring repeats.)
- If a table of random numbers is used without adequate specification of the numbers to be used (for example, 0001 to 2,000 or a different set of 2,000 four-digit numbers), the response is scored as incorrect (I).
- If a table of random numbers is used with adequate specification of the numbers to be used, as described above, a statement about ignoring repeats *AND* a statement about ignoring numbers not in the specified range are both needed to earn an E. If either or both are missing, the response is scored as partially correct (P). (The statement about ignoring repeats or the statement about ignoring numbers not in the range of specification may be implicit. For example, "... continue this procedure until 100 students are selected.")
- The procedure of using names or numbers on slips of paper in a hat must indicate some randomization in selection (for example, mixing the slips of papers or randomly choosing from the hat).
- If the sampling procedure does not produce a simple random sample, then the response is scored as incorrect (I).

# Part (b) is scored as follows:

Essentially correct (E) if the correct calculation is performed AND sufficient work is shown.

Partially correct (P) if the correct answer is provided but no work is shown.

Incorrect (I) if the response does not meet the criteria for E or P.

# Notes

- The correct formula (either in symbols or with correct numerical substitutions) is sufficient for shown work in either part (b) or part (c).
- A response with only the answer 0.413 with no work shown is scored as P (because dividing by the square root of 100 can be done mentally).

# Question 6 (continued)

- Sufficient shown work with no calculation is scored as partially correct (P). For example, showing  $\frac{4.13}{\sqrt{100}}$  with no calculation.
- Minor calculation errors can be ignored in part (b) or part (c) and considered when determining between two scores.
- The incorrect use of the notation  $\sigma$  instead of s is considered a minor error in the investigative task and will not reduce the score in either part (b) or part (c).

# **Part (c)** is scored as follows:

Essentially correct (E) if the response includes the following three components:

- 1. Combining variances.
- 2. Using weights (0.6 for females and 0.4 for males).
- 3. Recognizing variability of sample means (variance divided by sample size),

AND

the response correctly combines these three components to produce the estimated standard deviation for Rania's point estimator.

Partially correct (P) if the response has at least two of the three components *AND* a reasonable attempt to combine these components to produce the (overall) estimated standard deviation for Rania's point estimator.

Incorrect (I) if the response does not meet the criteria for E or P.

### Notes

- When part (c) is scored partially correct (P), the reasonableness of the numerical result can be considered in determining the overall score.
- The following are algebraically equal and provide the correct estimated standard deviation of Rania's point estimator of 0.198:

$$\sqrt{(0.6)^2 \frac{s_f^2}{n_f} + (0.4)^2 \frac{s_m^2}{n_m}} = \sqrt{(0.6)^2 \frac{s_f^2}{60} + (0.4)^2 \frac{s_m^2}{40}} = \sqrt{(0.6) \frac{s_f^2}{100} + (0.4) \frac{s_m^2}{100}} = \sqrt{\frac{(0.6)s_f^2 + (0.4)s_m^2}{100}} = \sqrt{\frac{(0.6)s_f^2 + (0.4)s_m^2}{100}}} = \sqrt{\frac{(0.6)s_f^2 + (0.4)s_m^2}{100}}} = \sqrt{\frac{(0.6)s_f^2 + (0.4)s_m^2}{100}} = \sqrt{\frac{(0.6)s_f^2 + (0.4)s_m^2}{100}} = \sqrt{\frac{(0.6$$

• A response that combines  $s_f^2$  and  $s_m^2$  to obtain a pooled standard deviation for Rania's data, such as

$$s_p = \sqrt{\frac{(n_f - 1)s_f^2 + (n_m - 1)s_m^2}{(n_f - 1) + (n_m - 1)}} = \sqrt{\frac{(59)(1.8)^2 + (39)(2.22)^2}{59 + 39}} \approx 1.97786,$$

contains two of the three components: combining variances and using weights

$$\left(\frac{59}{98} \approx 0.6 \text{ and } \frac{39}{98} \approx 0.4\right)$$
 for part (c). Because pooling is a reasonable attempt to combine the

components, the calculation of  $s_p \approx$  1.97786 is scored as partially correct (P). If the response

includes the correct third component, 
$$\left(\frac{s_p}{\sqrt{n}} \approx \frac{1.97786}{\sqrt{100}} = 0.197786\right)$$
, the response is still scored

partially correct, because the pooling is a reasonable, but not the correct, combination of the three components to compute the sample standard deviation of the stratified sample mean.

# Question 6 (continued)

# Part (d) is scored as follows:

Essentially correct (E) if a reasonable justification is provided based on *BOTH* of the following two components:

- 1. Smaller variability in responses for each gender in comparison with the variability in responses in Peter's data, and
- 2. Linkage (either explicit or implicit) between the smaller variability in responses for each gender and producing a smaller estimated standard deviation for Rania's combined point estimator than Peter's point estimator.

Partially correct (P) if the benefit of the smaller variability in responses for each gender with stratification (homogeneity within each gender) for these samples is identified, but there is no reference to how it affects the estimated standard error for Rania's point estimator,

OR

if the benefit of different centers with stratification (heterogeneity between genders) for the samples is identified, without reference to the resulting smaller variability in responses for each gender and how it affects the estimated standard error for Rania's point estimator,

OR

if the response identifies that Rania's combined data for males and females has smaller variability than Peter's data.

Incorrect (I) if the response does not meet the criteria for E or P.

# Notes

- Comments about shapes of the distributions are extraneous and can be ignored.
- General statements about the benefits of stratification without using the sample data in the dotplots are not sufficient.

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point.

- 4 Complete Response
- 3 Substantial Response
- 2 Developing Response
- 1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication, especially in the investigative parts—part (c) and part (d)—of the response.

# Scoring Worksheet

The following provides a worksheet and conversion table used for calculating a composite score of the exam.

# 2012 AP Statistics Scoring Worksheet

# **Section I: Multiple Choice**

# Section II: Free Response

# **Composite Score**

AP Score Conversion Chart Statistics

<u> </u>	
Composite	
Score Range	AP Score
70-100	5
57-69	4
44-56	3
33-43	2
0-32	1

<sup>\*</sup>Although 40 multiple-choice items were administered in Section I, item 14 was not used in scoring.

# **AP Statistics**

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